Veðurstofa Íslands
Report

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The Koch Index

Formulation, corrections and extension

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Introduction

The Koch-index is named after the Danish scientist Lauge Koch. In the years of the Second World War, he worked on representing graphically data on sea ice observed near the coasts of Iceland and calculating an index as a measure of ice extent, and published in 1945 [1]. These data were gathered from various sources, mostly by Icelandic geologist, Þorvaldur Thoroddsen (1916-1917) [2]. Koch's graphs extend for the time period from the first ages of settlement in Iceland to the year 1939. Later the series were extended, by Hlynur Sigtryggsson for the years 1940 to 1969 and by Eiríkur Sigurðsson for the years 1970 to 1983 [3], both at the Icelandic Meteorological Office. Also, the older series have been somewhat revised. Unfortunately, Lauge Koch did not leave any clear defininitions or description how exactly the index was calculated. Ever since, even though the basic idea behind it is well known, there have been some ambiguities how the index should be calculated and what should be taken into account.

This paper describes the latest extension of the Koch time series, made by the authors as a part of a Nordic climate research project, *Climate and sea-ice variability in the north Atlantic,* coordinated by dr. Martin Miles. Nansen Environmental and Remote Sensing Center, Bergen Norway. It contains full mathematical description how the index is calculated and corrects the values made by others, according to that description. The extension reaches from 1984 to 1990, and corrections have been made on the series from 1880 to 1983. So the Koch-series, consistent in the sense of our definitions now exist for the period from 1880 to 1990.

Some basic ideas

There are two different types of Koch series. One is the Simple-Koch series or just Koch-series. It is thought as the number of weeks sea-ice is observed near the coasts of Iceland. The other is the Complex-Koch series, which also takes into account the extent of the area in which ice is observed. It is calculated as the number of weeks sea ice is observed multiplied by the number of certain areas (see appendix B) ice is observed in. The Koch-indices have been calculated for the year as a whole, but also for certain time

intervals within each year. Thus seven different time series have been calculated. These are;

IW: The Simple Koch-index.

IW_1: The Simple Koch-index for the the time interval October-November-December.

IW_2: The Simple Koch-index for the the time interval January-February-March.

IW_3: The Simple Koch-index for the the time interval April-May-June.

IW_4: The Simple Koch-index for the the time interval July-August-September.

IW_{NOA}: The Simple Koch-index for the the time interval November-December-January-February-March.

IWR: The Complex Koch-index, only calculated for the year as a whole.

Note: The time interval "year", mentioned above refers to the sea ice year which consists of January-February-March-April-May-June-July– August-September for the actual year, but October-November-December for the year before.

Example:

The year 1970 or equally the sea ice year 1970 consists of January-February-March-April-May-June-July-Agust-September for the actual year 1970, but October-November-December for the actual year of 1969.

Formulating the calculation of the Koch-indices

The following definitions apply to a particular year:

$DayMax$: Number of days in that year.

Day : Any day-number that year. $1 \leq Day \leq DayMax$

f : Is the maximum distance from shore where sea ice contributes to the Koch index. To make the concerning indices more consistent through time, this distance must reduce with time. From 1940 to the current time, f is 20 nautical miles. Before 1940 f was much larger and undefined. This is because the modern methods of observation increase the number of observations and the area on which an observation occurs. The reference points from which f is measured, are listed in appendix B and also shown on a map.

$N_{d,f}$: The total number of days any sea ice is observed within the distance f from shore.

$ObsDay_i$: The day-numbers any sea ice is observed within the distance f from shore. $1 \le ObsDay_i \le DayMax$, $1 \le i \le N_{d,f}$.

$Month$: Any month of the sea ice year. $1 \le Month \le 12$

$Month_D$: The month of the sea ice year a particular *Day* belongs to. $1 \le Month_D \le 12$. October is no. 1, etc.

N_A: Number of areas on which sea ice is observed that year, within the distance f from shore.

The various Koch indices are defined as:

$$IW_1 = \frac{1}{7} \sum_{Day=1}^{DayMax} \sum_{i=1}^{N_{d,f}} \sum_{Month=1}^{3} \delta(Month, Month_D)\delta(Day, ObsDay_i) \quad (1)$$

$$IW_2 = \frac{1}{7} \sum_{Day=1}^{DayMax} \sum_{i=1}^{N_{d,f}} \sum_{Month=4}^{6} \delta(Month, Month_D)\delta(Day, ObsDay_i) \quad (2)$$

$$IW_3 = \frac{1}{7} \sum_{Day=1}^{DayMax} \sum_{i=1}^{N_{d,f}} \sum_{Month=7}^{9} \delta(Month, Month_D)\delta(Day, ObsDay_i) \quad (3)$$

$$IW_4 = \frac{1}{7} \sum_{Day=1}^{DayMax} \sum_{i=1}^{N_{d,f}} \sum_{Month=10}^{12} \delta(Month, Month_D)\delta(Day, ObsDay_i) \quad (4)$$

The indices i, in IW_i, i=1,2,3,4 refer to each quarter of the sea-ice year.

$$IW_{NOA} = \frac{1}{7} \sum_{Day=1}^{DayMax} \sum_{i=1}^{N_{d,f}} \sum_{Month=2}^{6} \delta(Month, Month_D)\delta(Day, ObsDay_i) \quad (5)$$

$$IW = \frac{1}{7} \sum_{Day=1}^{DayMax} \sum_{i=1}^{N_{d,f}} \delta(Day, ObsDay_i) = \sum_{i=1}^{4} IW_i = \frac{1}{7} N_{d,f} \quad (6)$$

$$IWR = IW \cdot N_A \quad (7)$$

where $\delta(a,b)$ is the Kronecker-delta function.

The corrections

Because there do not exist any actual numbers for the latest revision for the time interval 1880 to 1975, it has been necessary to just read the values from existing bar-figures, see the figure 1 page 5. This introduces an error, both from the scientist who has drawn the columns and from the scientist who

reads the values later on. With this in mind and also due to some inconsistency in making the old values, a correction has been made in accordance with the above text. Following is an example on how the corrections are made.

	Read values from table.						Corrected values from table.						
Year	IW1	IW2	IW3	IW4	IW	IWR	IW1	IW2	IW3	IW4	IW	IWR	#A
1880	5,38	0,00	0,00	0,00	5,38	9,06	5,43	0,00	0,00	0,00	5,43	10,86	2
1881	0,00	12,86	8,71	0,00	21,57	161,23	0,00	12,86	8,71	0,00	21,57	151,00	7
1882	0,00	0,00	10,84	9,95	20,78	108,70	0,00	0,00	10,86	10,00	20,86	104,29	5
1908	0,00	0,00	2,40	0,00	2,40	3,07	0,00	0,00	2,43	0,00	2,43	2,43	1
1909	0,00	0,00	0,71	0,00	0,71	0,00	0,00	0,00	0,71	0,00	0,71	0,71	1
1974	0,92	0,37	0,00	0,00	1,29	6,13	0,86	0,43	0,00	0,00	1,29	6,43	5
1975	0,00	0,00	3,64	6,27	9,91	28,22	0,00	0,00	3,57	6,29	9,86	29,57	3

Table 1: A few values from the actual table as an example. Here #A equals N_A.

In table 1, the read value of first quarter of year, IW_1 is multiplied by 7 and the outcome is rounded to the nearest integer. This is in accordance with equation (1) above. Then the outcome is again divided by 7 to get the corrected IW1 value. The same procedure applies to the second, third and fourth quarter of year. Summing up the numbers IW_1, IW_2, IW_3 and IW_4 makes the corrected IW-value. This is done according to equation (6) above.

In accordance with equation (7), the number of observed areas N_A is gained by dividing read-IWR value with read-IW value and then rounding the outcome to the nearest integer. This number is then multiplied by the corrected IW value to get the corrected IWR. The difference between the read and corrected IWR value is minor.

The corrected values are listed in appendix A.

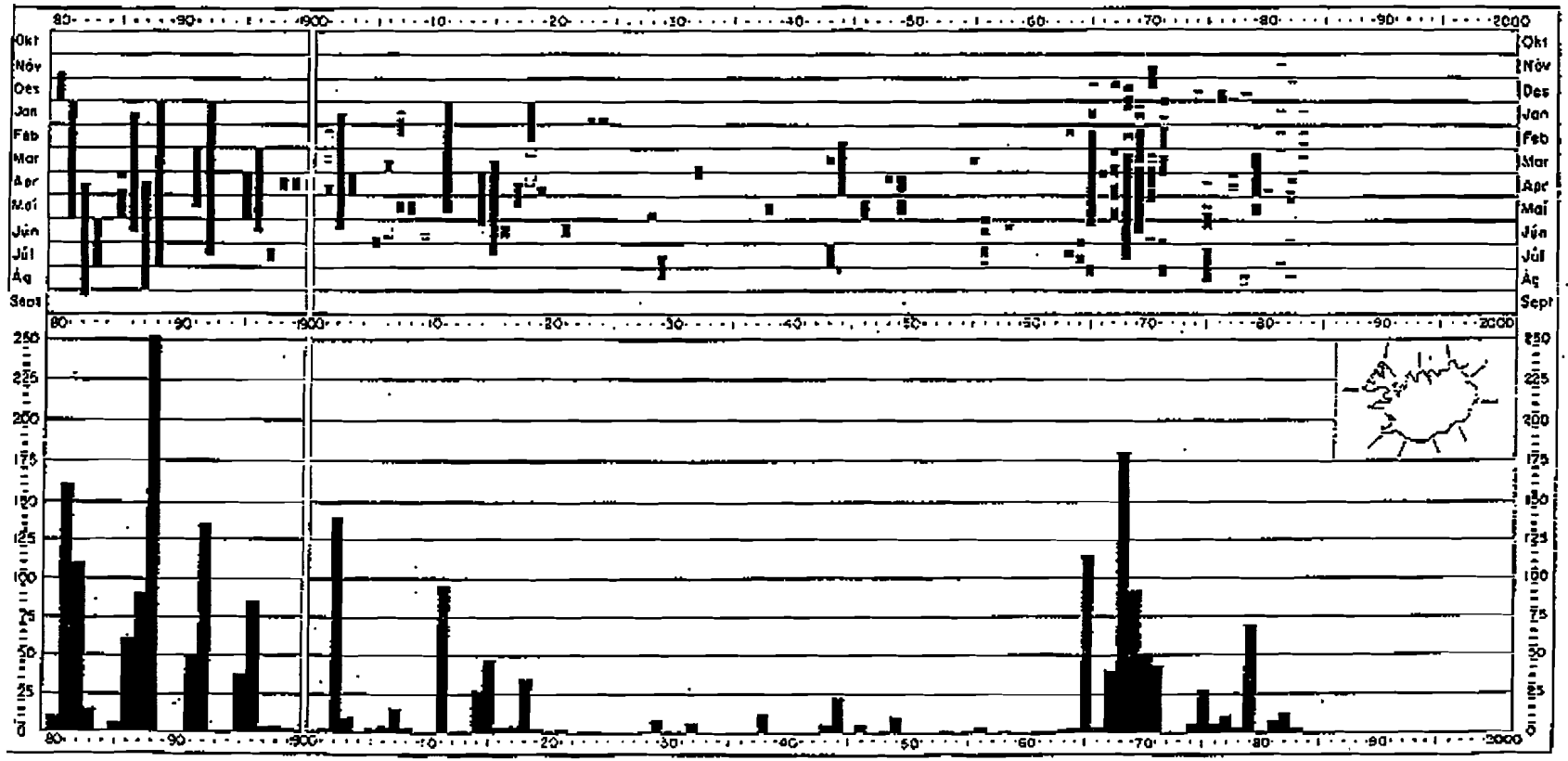


Figure 1. The Koch series from 1880 to 1983 as represented by Staðarvalsnefnd (1986).

References

[1] *Koch, L.* 1945: The East Greenland Ice. Meddelelser fra Grönland. Bd. 130 nr. 3, Köbenhavn.

[2] *Thoroddsen, Th.* 1916-1917: Árferði á Íslandi í þúsund ár. Kaupmannahöfn.

[3] *Staðarvalsnefnd,* 1986. Staðarval fyrir álver. Lokaskýrsla. Iðnaðarráðuneytið, Reykjavík í júlí 1986.

Appendix A

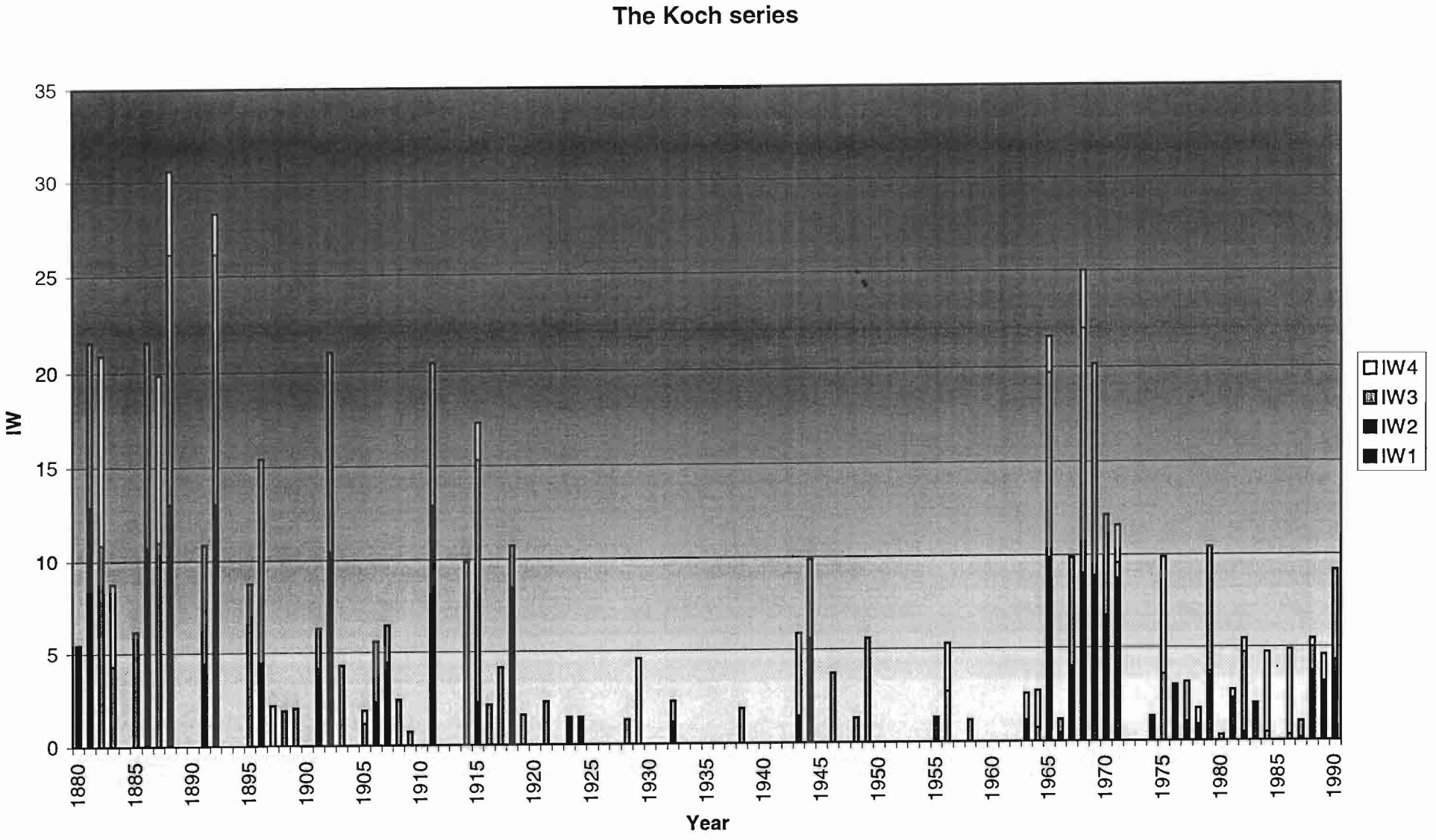
Folowing are the final results.

Year	IW_1	IW_2	IW_3	IW_4	IW	IWR	N_A	IW_{NAO}
1880	5,43	0,00	0,00	0,00	5,43	10,86	2	5,43
1881	0,00	12,86	8,71	0,00	21,57	151,00	7	12,86
1882	0,00	0,00	10,86	10,00	20,86	104,29	5	0,00
1883	0,00	0,00	4,29	4,43	8,72	17,29	2	0,00
1884	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1885	0,00	0,00	6,14	0,00	6,14	6,14	1	0,00
1886	0,00	10,71	10,86	0,00	21,57	64,71	3	10,71
1887	0,00	0,00	11,00	8,86	19,86	79,43	4	0,00
1888	0,00	13,00	13,14	4,43	30,57	244,57	8	13,00
1889	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1890	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1891	0,00	4,43	6,43	0,00	10,86	54,29	5	4,43
1892	0,00	13,00	13,14	2,14	28,28	141,43	5	13,00
1893	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1894	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1895	0,00	0,00	8,71	0,00	8,71	34,86	4	0,00
1896	0,00	4,43	11,00	0,00	15,43	92,57	6	4,43
1897	0,00	0,00	0,00	2,14	2,14	4,29	2	0,00
1898	0,00	0,00	1,86	0,00	1,86	3,71	2	0,00
1899	0,00	0,00	2,00	0,00	2,00	2,00	1	0,00
1900	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1901	0,00	4,14	2,14	0,00	6,28	6,29	1	4,14
1902	0,00	10,43	10,57	0,00	21,00	147,00	7	10,43
1903	0,00	0,00	4,29	0,00	4,29	8,57	2	0,00
1904	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1905	0,00	0,00	1,14	0,71	1,85	1,86	1	0,00
1906	0,00	2,29	3,29	0,00	5,58	5,57	1	2,29
1907	0,00	4,43	2,00	0,00	6,43	12,86	2	4,43
1908	0,00	0,00	2,43	0,00	2,43	2,43	1	0,00
1909	0,00	0,00	0,71	0,00	0,71	0,71	1	0,00
1910	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1911	0,00	12,86	7,57	0,00	20,43	102,14	5	12,86
1912	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1913	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1914	0,00	0,00	9,86	0,00	9,86	29,57	3	0,00
1915	0,00	2,29	13,00	2,00	17,29	51,86	3	2,29
1916	0,00	0,00	2,14	0,00	2,14	4,29	2	0,00
1917	0,00	0,00	4,14	0,00	4,14	4,14	1	0,00
1918	0,00	8,43	2,29	0,00	10,72	31,14	2,9	8,43
1919	0,00	0,00	1,57	0,00	1,57	3,14	2	0,00
1920	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1921	0,00	0,00	2,29	0,00	2,29	2,29	1	0,00
1922	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1923	0,00	1,43	0,00	0,00	1,43	1,43	1	1,43
1924	0,00	1,43	0,00	0,00	1,43	1,43	1	1,43
1925	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1926	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1927	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1928	0,00	0,00	1,29	0,00	1,29	1,29	1	0,00
1929	0,00	0,00	0,00	4,57	4,57	9,14	2	0,00
1930	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00

1931	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1932	0,00	1,14	1,14	0,00	2,28	6,86	3	1,14
1933	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1934	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1935	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1936	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1937	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1938	0,00	0,00	1,86	0,00	1,86	13,00	7	0,00
1939	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1940	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1941	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1942	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1943	0,00	1,43	0,00	4,43	5,86	5,86	1	1,43
1944	0,00	5,57	4,29	0,00	9,86	19,71	2	5,57
1945	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1946	0,00	0,00	3,71	0,00	3,71	3,71	1	0,00
1947	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1948	0,00	0,00	1,29	0,00	1,29	1,29	1	0,00
1949	0,00	0,00	5,57	0,00	5,57	11,14	2	0,00
1950	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1951	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1952	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1953	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1954	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1955	0,00	1,29	0,00	0,00	1,29	1,29	1	1,29
1956	0,00	0,00	2,71	2,56	5,27	5,29	1	0,00
1957	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1958	0,00	0,00	1,14	0,00	1,14	2,29	2	0,00
1959	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1960	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1961	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1962	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1963	0,00	1,14	0,00	1,43	2,57	2,57	1	1,14
1964	0,00	0,00	0,71	2,00	2,71	2,71	1	0,00
1965	0,71	9,14	9,86	1,86	21,57	107,86	5	9,85
1966	0,00	0,43	0,71	0,00	1,14	3,43	3	0,43
1967	1,43	2,57	5,86	0,00	9,86	39,43	4	4,00
1968	2,29	6,71	13,00	3,00	25,00	175,00	7	9,00
1969	0,00	8,86	11,29	0,00	20,15	100,71	5	8,86
1970	4,00	2,71	5,43	0,00	12,14	48,57	4	6,71
1971	0,57	8,14	0,86	2,00	11,57	46,29	4	8,71
1972	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1973	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1974	0,86	0,43	0,00	0,00	1,29	6,43	5	1,29
1975	0,00	0,00	3,57	6,29	9,86	29,57	3	0,00
1976	2,14	0,71	0,00	0,14	2,99	6,00	2	2,85
1977	0,57	0,43	2,14	0,00	3,14	12,57	4	1,00
1978	0,86	0,00	0,00	0,86	1,72	3,43	2	0,86
1979	0,00	3,71	6,71	0,00	10,42	73,00	7	3,71
1980	0,00	0,00	0,29	0,00	0,29	0,29	1	0,00
1981	0,71	1,43	0,14	0,43	2,71	10,86	4	2,14
1982	0,29	0,14	4,29	0,71	5,43	16,29	3	0,43
1983	0,14	1,71	0,14	0,00	1,99	4,00	2	1,85
1984	0,00	0,00	0,43	4,29	4,72	14,14	3	0,00
1985	0,00	0,00	0,00	0,00	0,00	0,00	0	0,00
1986	0,00	0,00	0,29	4,57	4,86	14,57	3	0,00

1987	0,00	0,14	0,86	0,00	1,00	2,00	2	0,14
1988	0,00	3,71	1,29	0,43	5,43	27,14	5	3,71
1989	0,29	2,86	0,00	1,43	4,58	9,14	2	3,15
1990	3,57	0,71	4,86	0,00	9,14	27,43	3	4,28

Figures 2 through 4 show graphs of the various Koch series.


The Koch series
IW
Year
IW4
IW3
IW2
IW1
0
5
10
15
20
25
30
35
1880
1885
1890
1895
1900
1905
1910
1915
1920
1925
1930
1935
1940
1945
1950
1955
1960
1965
1970
1975
1980
1985
1990

Fgure 2.

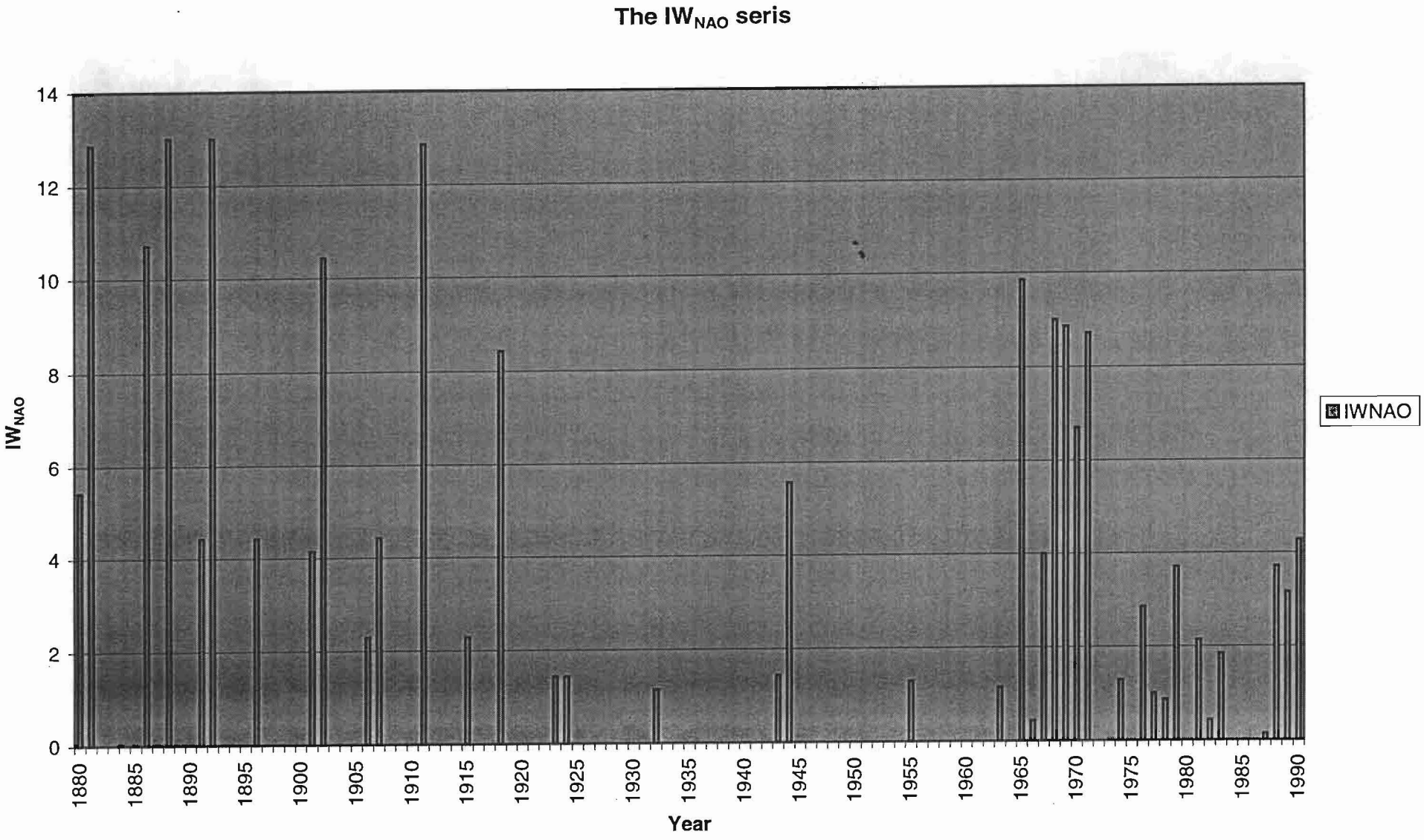

The IW$_{NAO}$ seris
IW$_{NAO}$
14
12
10
8
6
4
2
0
1880
1885
1890
1895
1900
1905
1910
1915
1920
1925
1930
1935
1940
1945
1950
1955
1960
1965
1970
1975
1980
1985
1990
Year
IWNAO

Figure 3.

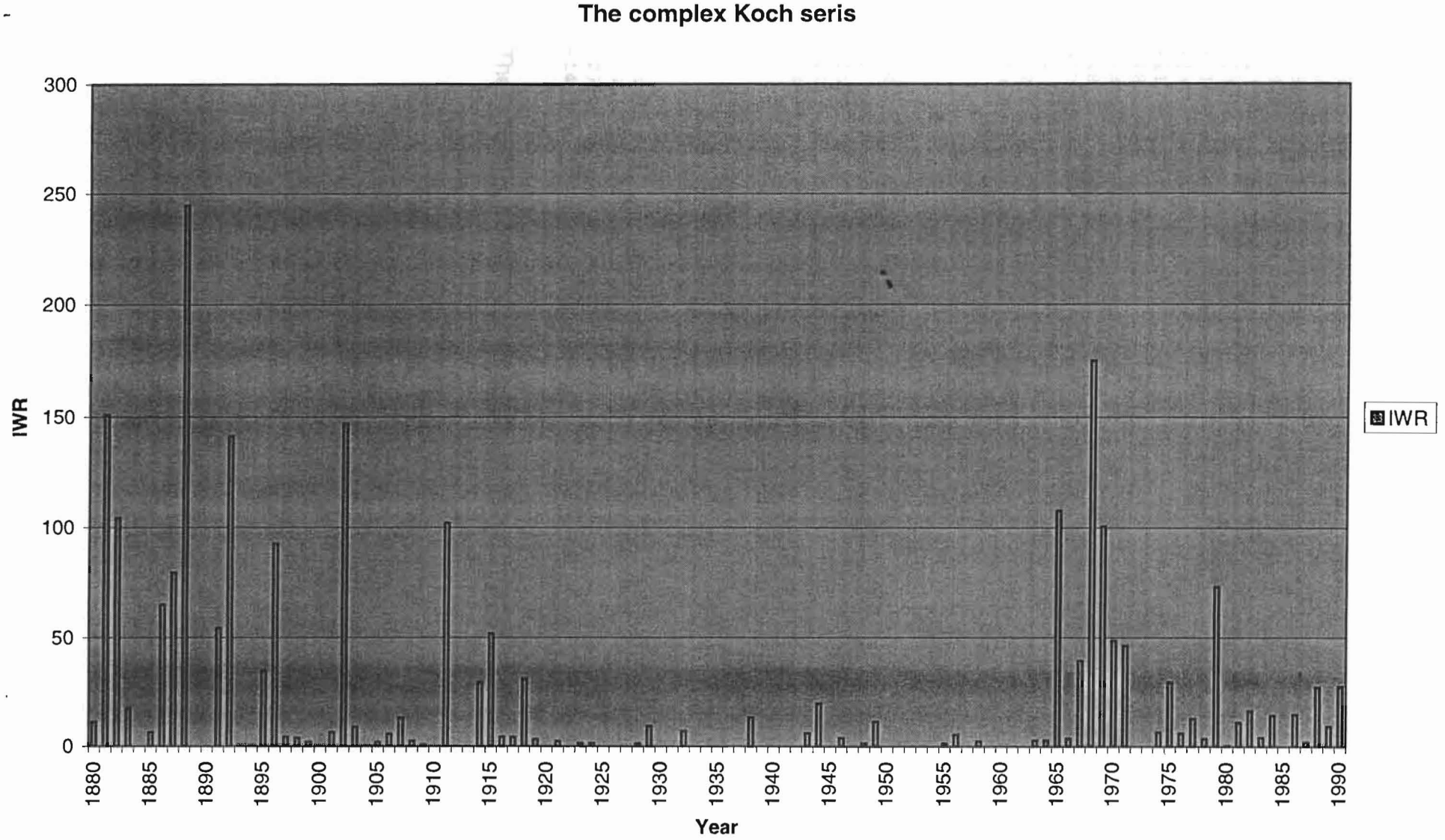

The complex Koch seris
300
250
200
150
100
50
0
IWR
1880
1885
1890
1895
1900
1905
1910
1915
1920
1925
1930
1935
1940
1945
1950
1955
1960
1965
1970
1975
1980
1985
1990
Year
IWR

Figure 4

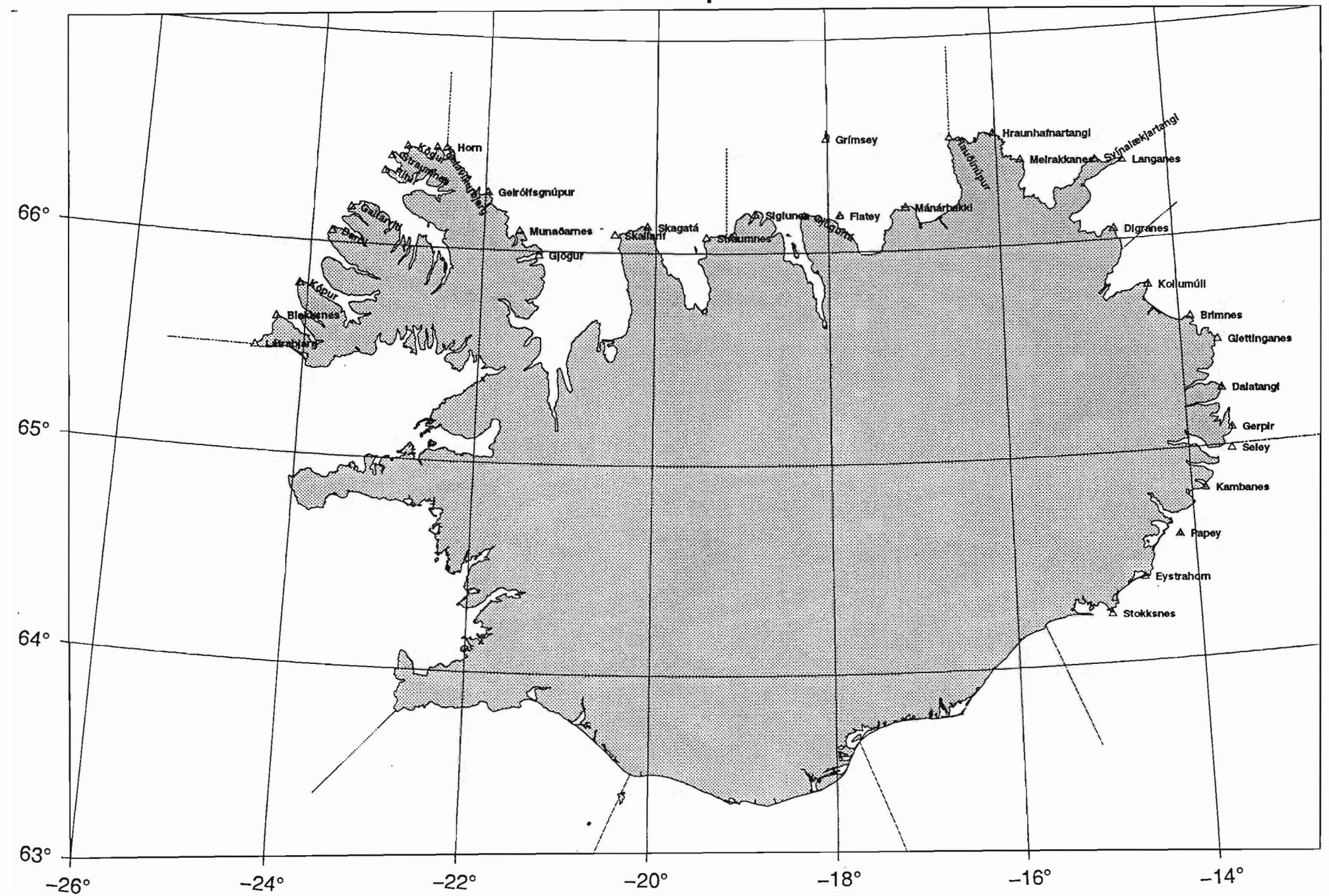
Appendix B

Following are the reference points from which f is measured, also shown on map 1.

-23.77	66.05	Barði
-24.55	65.50	Bjargtangar
-24.33	65.64	Blakksnes
-14.68	66.05	Digranes
-13.58	65.27	Dalatangi
-14.59	64.40	Eystrahorn
-17.87	66.17	Flatey
-23.57	66.17	Galtarviti
-21.97	66.27	Geirólfsgnúpur
-13.50	65.08	Gerpir
-21.35	65.98	Gjögur
-18.28	66.17	Gjögur
-13.58	65.50	Glettinganes
-18.02	66.53	Grímsey
-22.48	66.47	Horn
-16.03	66.53	Hraunhafnartangi
-22.60	66.47	Hælavíkurbjarg
-13.85	64.80	Kambanes
-14.33	65.78	Kollumúli
-24.10	65.80	Kópur
-22.95	66.47	Kögur
-13.87	65.62	Kögur
-14.53	66.37	Langanes
-17.10	66.20	Mánárbakki
-15.72	66.40	Melrakkanes
-21.58	66.09	Munaðarnes
-14.17	64.59	Papey
-16.55	66.52	Rauðinúpur
-23.20	66.35	Ritur
-13.52	64.98	Seley
-18.85	66.18	Siglunes
-20.10	66.12	Skagatá
-20.48	66.08	Skallarif
-14.97	64.23	Stokksnes
-19.41	66.07	Straumnes
-23.13	66.42	Straumnes
-14.85	66.38	Svínalækjartangi

These points and the different areas are shown on Map 1.

Map 1


Horn
Grímsey
Hraunhafnartangi
Melrakkanes
Langanes
Geirólfsgnúpur
Munaðarnes
Skagatá
Skaftanes
Siglunes
Flatey
Mánárbakki
Digranes
Gjögur
Kollumúli
Brimnes
Glettinganes
Dalatangi
Gerpir
Seley
Kambanes
Papey
Eystrahorn
Stokksnes
66°
65°
64°
63°
−26°
−24°
−22°
−20°
−18°
−16°
−14°