Veðurstofa Íslands
Report

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A Method for Avalanche Risk Assessment
Short description

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1. Introduction

This report explains shortly the main ingredients in a hazard zoning method that was first developed at the University of Iceland (UI) in 1995–1996 and then both used and further developed at the Icelandic Meteorological Office (IMO) since September 1996. The work was initiated in reaction to the avalanche which fell on the town Súðavík in January 1995 and killed 14 people. Most of the houses hit were in an area that was marked "safe" on the official avalanche hazard map. Work on the method was escalated when an avalanche fell on the town Flateyri in October 1995 killing 20 people, all in the "safe" area.

The method is a risk assessment method for housing areas. The aim is to measure the avalanche hazard by calculating the probability of being killed in an avalanche if one lives or works at the place under consideration for a given length of time. In short the method is based on estimating the frequency of avalanches locally, in each avalanche path separately, or jointly in a few adjacent "comparable" paths. The runout distance distribution, i.e. the relative probability of different runout distances, is on the other hand estimated using data from many avalanche paths in different parts of the country. To facilitate this global estimate, a measuring scale for runout has been developed, through which each avalanche and in fact each position under a hillside can be assigned a *runout index* ("rennslisstig" in Icelandic). This scale enables the *transfer* of avalanches between paths. The runout index of an avalanche may be interpreted as the runout distance of the avalanche measured in hectometres after it has been transferred to a certain standard slope.

Having estimated the frequency and the runout distance distribution the remaining major ingredient in the risk assessment is to estimate the probability that a person survives if he is staying in a house that is hit by an avalanche travelling at a given speed. This probability has been estimated using data from the avalanches of Súðavík and Flateyri. These avalanches damaged 32 houses where 93 people were staying. As mentioned abeve, 34 of these were killed.

The method is best suited to calculating the risk due to avalanches under hillsides that have some history of avalanches. It may also be used for setting an upper limit on the risk when there are no recorded avalanches.

The people involved in the development of the method are and were Kristján Jónasson (now at IMO but at UI before), Þorsteinn Arnalds (IMO), Sven Þ. Sigurðsson (UI), Gunnar G. Tómasson (UI) and Kristín Friðgeirsdóttir (UI, but now at Stanford University). In the beginning the aim of the project work at the University was to estimate the return periods of avalanches but soon the estimation of risk was incorporated. In October 1995 a progress report was written (in Icelandic). Following the avalanche in Flateyri it was decided to make *runout index maps* for the towns where the avalanche hazard was considered to be greatest. The maps were accompanied by a report (also in Icelandic). The final report of the University project is now being written and this will (hopefully) contain many details that we will omit here.

In the next chapter we discuss risk and its measurement, address the question of what level of risk should be deemed acceptable and mention the connection between risk and return periods of avalanches. In chapter 3 the idea of transferring avalanches between paths and the concept of runout index are developed. The next three chapters discuss the three main ingredients in the risk calculation, i.e. the runout distance distribution, the survival probability and avalanche frequency. Chapter 7 contains a short discussion of what we have termed "tongue effect". Finally we tie the strings together in the 8th chapter and present formulas for calculating avalanche risk. The final chapter also contains a short overview of the experience we have had with the method.

2. Risk

2.1 Measures of risk.

Sometimes the word risk is used to mean "hazard that has been measured or quantified" and we will use this definition here. Before embarking on the measurement one must agree upon a unit to use for avalanche risk. There are several possibilities to define this unit. One might measure the return period of avalanches, the expected value of property lost in avalanches (economic risk), the expected number of people killed in the area in a given time period, and finally one can measure *individual risk* as the annual probability of being killed in an avalanche if one lives or works in a building under a hazardous hillside. The last definition is the chosen one, but to make it workable one must specify firstly the type of building and secondly the proportion of the time spent in the building. Most of the houses in the avalanche hazard towns in Iceland are quite weak timber or concrete houses with relatively large windows facing the mountain side and in the work presented here such a house is assumed. The risk is then calculated based on the person being present in the building 100% of the time.

2.2 Probability of being present

The probability of being killed is found by multiplying the calculated risk with an estimate of the probability that the person is at home or at work when the avalanche strikes. This "presence probability" depends on the age of the person and the type of the building. For living houses it might be as high as 75% for children but lower for adults. For work places it is lower than for houses, maybe about 30%, and it will be lower still in cottages (often less than 5%). This difference is the main reason for not including the presence probability in the calculated risk.

2.3 Acceptable risk

Associated with risk measurement is the concept of *acceptable risk*. Having estimated the risk at each spot in a given area the risk value that is considered acceptable will define the limits of the hazard zones. The common method of determining the acceptable risk level due to a particular hazard is to compare it with other risks. The acceptable level of avalanche risk has been much discussed in Iceland and at the moment the IMO is using the value $\mathbf{0.3 \cdot 10^{-4}}$ for living houses. For Icelandic children aged 1–15 years the yearly death rate from all causes is approximately $2 \cdot 10^{-4}$, about half of this due to accidents and the other half due to illness. About 40% of the accidents are traffic accidents. Assuming a 75% presence probability the avalanche hazard on the acceptable risk line will add $0.225 \cdot 10^{-4}$ or 11% to the death rate of children. In areas carrying this risk the expected number of children killed in avalanches will be about half of the expected number of traffic victims. This comparison assumes that children are as likely as adults to be killed by avalanches, which has been confirmed in Icelandic avalanche accidents. For work places the IMO has been using an acceptable risk level of $\mathbf{1 \cdot 10^{-4}}$ and this figure has been justified by a similar comparison as the acceptable level for houses. It is higher both because the presence probability for work places is lower than for homes and because the death rate for adults is higher than for children. For summer cottages a level of $\mathbf{5 \cdot 10^{-4}}$ has been used. We summarise these numbers along with a few added details in Table 1 below.

Type of building	Acceptable risk	Deciding age group	Annual death rate	Presence probability	Increase in death rate due to avalanche hazard: absolute	relative	Assumption behind death rate increase
Living house	$0.3 \cdot 10^{-4}$	1-15 yrs	$2 \cdot 10^{-4}$	75%	$0.22 \cdot 10^{-4}$	11%	The school is safe
Work place	$1 \cdot 10^{-4}$	15-30 yrs	$7 \cdot 10^{-4}$	30%	$0.48 \cdot 10^{-4}$	7%	The home is on a $0.3 \cdot 10^{-4}$ line and 60% of the time is spent there
Summer cottage	$5 \cdot 10^{-4}$	1-15 yrs	$2 \cdot 10^{-4}$	5%	$0.25 \cdot 10^{-4}$	12%	The home and the school are safe

Table 1. Determination of acceptable risk.

2.4 *Return periods of avalanches*

In Norway regulations state that new houses shall not be built where avalanches fall more frequently than once every 1000 years. In Switzerland the limit of the hazard zone is set at the tip of a 300 year avalanche but these return periods are not entirely comparable to the Norwegian return periods. Firstly the avalanche may go past the house without hitting it and secondly the runout distance of the 300 year avalanche is in practice calculated from an estimated 300 year maximum of 3 day snow fall in the starting zones and this extreme snow fall does not necessarily produce an avalanche each time. In fact there are some grounds to believe that the actual limits of the hazard zones both in Norway and in Switzerland correspond to a return period that is somewhat higher than 1000 years (meaning that a house on the limits will be hit by an avalanche more seldom than once every 1000 years). According to table 6 in chapter 8 the $0.3 \cdot 10^{-4}$ risk line as given by the method described here corresponds on average to the 5000 year line. This is however quite variable depending on the frequency of avalanches and can range from 2000 years for the hillsides with most frequent avalanches to 6000 years or higher for the hillsides where avalanches fall seldom. This indicates that return periods are not a good unit for measuring risk.

Note however that this variability is dependent on the shape of the runout index distribution (see chapter 4) and might be less marked for a distribution that is based on a different data set. The variability will also be reduced if there are still missing avalanches at runout index 16 (see section 4.4). If the basic assumption at the beginning of section 4.3 does not hold so that a low frequency implies relatively less probability of extremely long avalanches then the difference will also be less marked.

3. Runout scales

3.1 *Transferring avalanches between paths*

One of the major tasks in hazard zoning is the estimation of the possible runout distances of avalanches that come very seldom indeed as the numbers of the previous section indicate. Avalanche records in Iceland only go about 100 years back in time and therefore it is impossible to base the estimation of the frequency of long avalanches that come every several thousand years on local history alone. By combining the avalanche history of many paths one may however imagine that one path has been observed for a long time rather than many paths for a short time. To make this possible, some kind of a scale for the runout of avalanches is needed, that would tell how far an avalanche that has fallen in a given path would reach in another path. In other words one needs a method that enables the *transfer* of avalanches between paths.

The Norwegian alpha-beta model may be considered as an example of such a transfer method. This method has been described in several reports and articles. Tómas Jóhannesson at the IMO has developed an alpha beta-model for Icelandic avalanches. A similar model has been developed for Austrian avalanches. The runout ratio of McClung can also be thought of as a transfer method.

As mentioned in the introduction we have developed a different scale for measuring runout, *runout indices*. The runout index of an avalanche is calculated using a version of the well known PCM-model. One of the advantages of using the PCM model rather than the alpha-beta model is that the former enables the estimation of the speed of the avalanche and we shall see that this is an essential feature. The next section discusses the use of this model.

3.2 *The PCM model*

Several physical models have been used to simulate avalanches and one oft the better known is the PCM model. Two parameters enter the model, a friction parameter, μ, and an inverse drag coefficient, M/D. The avalanche is described by specifying how its centre of mass will travel down the hillside. According to the model two avalanches are the same if the values of μ and M/D are the same. The determination of the input parameters is not easy as there are an infinite number of pairs that can explain a given avalanche runout in a given path. Increased friction can be compensated for by decreased drag (increased inverse drag). The curves in Figure 1 represent the different pairs of coefficients that can explain a few Icelandic avalanches.

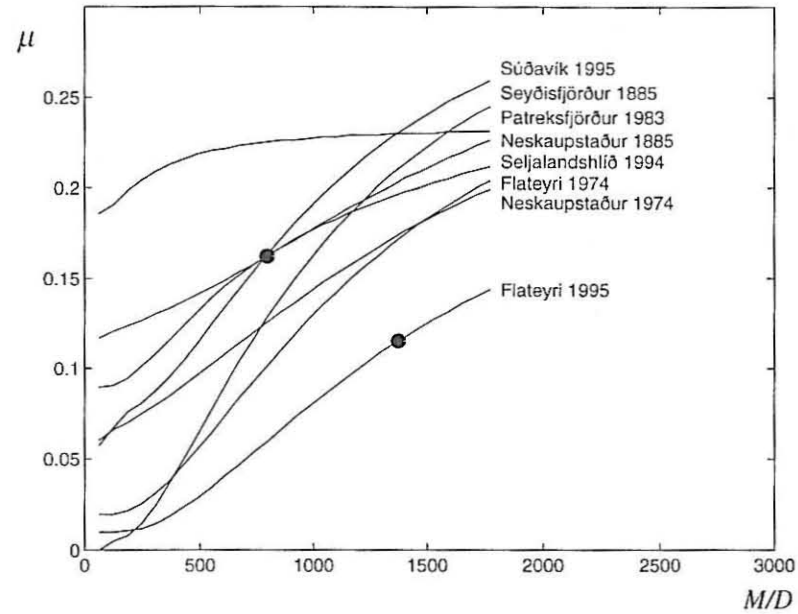

Figure 1. Isorunlines for a few well known avalanches. The black spots correspond to avalanches with known fracture height.

By assuming that the input parameters of the PCM model obey a probability distribution that is independent of the path it is possible to use the model for the transfer of avalanches between paths. Assume that the (μ, *M/D*) pair is a random variable that lies in the region defined by $0.05 \leq \mu \leq 0.6$, $40 \leq M/D \leq 4000$ and obeys a bivariate normal distribution inside this square. To be precise the joint density function of (μ, *M/D*) is assumed to be zero outside the rectangle and a multiple of a bivariate normal distribution inside it, the multiplier chosen so that the total probability is 1.

By calculating the curves of possible parameter pairs (*isorunlines*) for each avalanche in some data set of avalanches, it is possible to use the method of maximum likelihood to calculate the most probable density function of this type. Figure 2 shows the isorunlines for 197 Icelandic avalanches along with the contour lines of the estimated density function (the ellipses). In the maximum likelihood estimation the fact that some of the avalanches fell into the sea has been taken into account in an appropriate way. For about 10 of the avalanches the height of the fracture line is recorded. For these avalanches it is possible to estimate the most likely parameter pair (by making some further assumptions) and the resulting pairs are shown by the filled points in Figures 1 and 2. The maximum likelihood estimate has taken the position of these points into account.

The longer an avalanche travels, the lower the friction and/or higher the inverse drag coefficient that is needed to explain its runout. Therefore the long avalanches have their isorunlines placed low or near the *M/D*-axis in the parameter space. It is seen in Figures 1 and 2 that the isorunlines intersect one another so it is not enough to know the isorunlines to enable the transfer of avalanches between paths. We have developed two ways to proceed. To transfer an avalanche that stopped at a given point to a different hillside, we can determine the point under the second hillside which has the same runout probability as the stopping point and say: "This is where the avalanche would probably have stopped if it had fallen here". The runout probability of a point is the probability that an avalanche will reach the point according to the density function of the parameters. It is given by the volume under the function between the isorunline of the point and the *M/D*-axis.

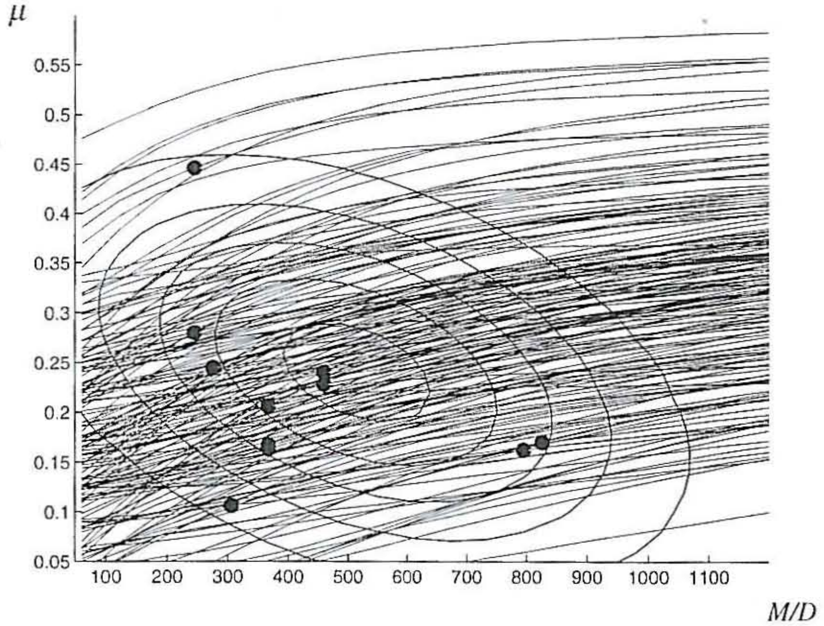

Figure 2. Isorunlines for 197 Icelandic avalanches and contours of the fitted bivariate normal density function. The black spots correspond to avalanches with known fracture height.

The second way to use the isorunlines to transfer avalanches is based on the fact that even if the isorunlines intersect they are approximately parallel. For each avalanche, the point on the corresponding isorunline that maximises the density function is in some sense the most probable value of (μ, *M/D*) that will explain the avalanche. We find an axis through the centre of the contour ellipses that approximates the collection of these point. For a given runout distance we determine the point where the corresponding isorunline intersects this axis. If the isorunline for a runout distance in a different hillside intersects the axis in the same point then the two runout distances are comparable and one can be transferred to the other. Figure 3 shows the contour ellipses, a selection of the isorunlines, the approximating axis and the most probable values of (μ, *M/D*) for all 197 avalanches.

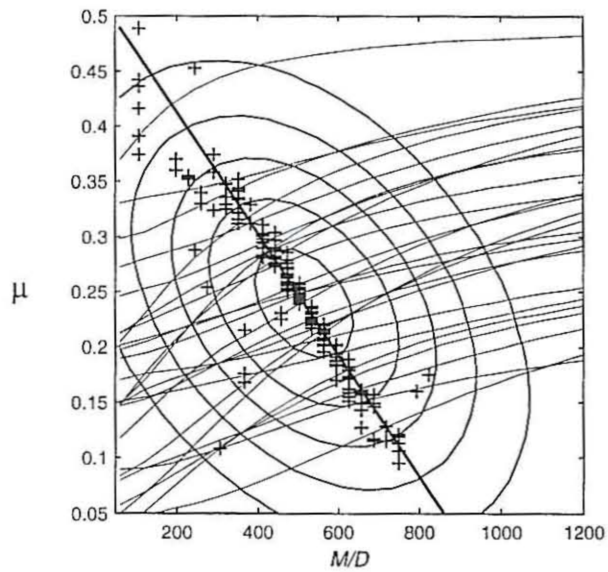

Figure 3. A few isorunlines, contour lines for the density function, most likely parameter pairs and the approximating axis.

It turns out that the two transfer methods discussed shortly above give very similar results. An advantage of the second method is that because each avalanche is assigned a particular pair of input values one may calculate the speed of the avalanche along the profile using the PCM model and this fact is used in the risk calculations discussed below.

3.3 The avalanche data set

The data on the 197 avalanches mentioned above were collected by Kristín Friðgeirsdóttir at the UI in 1995–1996. The avalanches fell from 49 different paths and the oldest ones fell just over 100 years ago. Some of the paths have a shorter observation history and it cannot be far off to guess that the average observation period is about 80 years. The data set contains (in computer files) 23 pieces of data about each avalanche, for instance the path name, the date, the stopping position and the width. Also recorded is the path profile. The data set is based on avalanche maps and accompanying avalanche lists obtained from the IMO covering 8 Icelandic towns and villages. All the avalanches shown on these maps were included in the data set. The maps covered the period up till 1989 but a few avalanches that fell later were also included.

3.4 Runout indices

To get a descriptive and uniform scale for measuring runout distance we have taken the course of defining a *standard slope* that is representative for the Icelandic avalanche paths (and to some extent Norwegian paths also). An avalanche can be transferred to the standard slope and the (horizontal) runout distance there measured in hectometres defines the *runout index* of the avalanche. The standard slope is shown in Figure 4. It is parabola shaped, 700 m high and reaches level ground 1600 m from the starting point. The equation for it is

$$y = \begin{cases} \dfrac{700}{1600^2}(x-1600)^2 & \text{if } 0 \le x \le 1600 \\ 0 & \text{if } x > 1600 \end{cases}$$

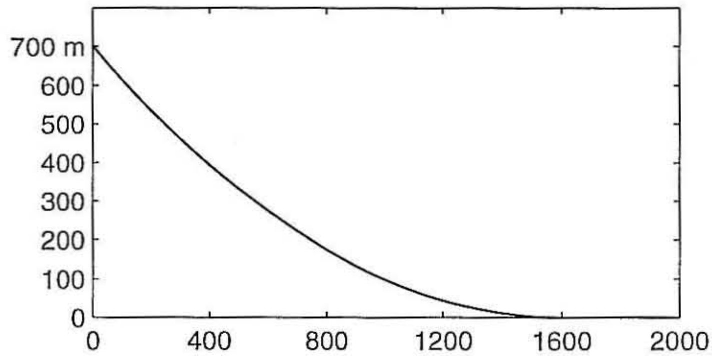

Figure 4. The standard slope.

To calculate the runout index of an avalanche one proceeds as follows: First the isorunline of the avalanche is calculated, by finding for a set of *M/D* values (for instance 50, 100, 150, ..., 1200) corresponding μ values, that will make the PCM-simulated avalanche stop in the correct place. The intersection of the isorunline and the axis-line discussed in section 3.2 determines a pair (μ, *M/D*). A PCM-simulated avalanche with this parameter pair is then set off at the top of the standard slope, and its stopping position gives the runout index. Table 2 contains a list of the 10 avalanches in the data set that have the highest runout index. We remark that (perhaps surprisingly) none of these avalanches fell into the sea.

Town	Path	Year	Runout index
Flateyri	Skollahvilft	1995	18.5
Flateyri	Innra-Bæjargil	1974	17.1
Flateyri	Skollahvilft	1953	16.8
Ísafjörður	Tunguskógur	1994	16.7
Neskaupstaður	Bakkagil	1974	16.5
Neskaupstaður	Innri Sultarbotnagjá	1936	16.5
Hnífsdalur	Búðarhyrna, Traðargil	1947	16.3
Neskaupstaður	Gully below Gunnólfsskarð	1990	16.1
Súðavík	Súðarvíkurhlíð	1995	16.1
Ísafjörður	Seljalandsdalur, old ski hut	1953	15.9

Table 2. The ten longest Icelandic avalanches.

4. Runout index distribution

4.1 Combined avalanche history

It was mentioned above that because avalanche records in Iceland do not go very far back it might prove advantageous to combine the avalanche history of many paths in order to be able to estimate the frequency of long avalanches. By transferring all the avalanches in the Icelandic data set to the standard slope we can imagine that we have a 4000 year observation period there instead of having watched 50 paths for 80 years. Continuing along this track it would now be possible to estimate the frequency of avalanches that reach a given runout index by counting the number of avalanches in the data set that have travelled farther. There are for instance 8 avalanches that have a runout index of 16 or higher giving an average return period of 4000/9 ≈450 years.

Such a direct calculation has however several flaws. Firstly, the overall frequency of avalanches in different hillsides is different. Secondly, even if the overall frequency in a path is high it does not necessarily follow that the frequency of long avalanches is also high. For some avalanche paths a high runout might simply be impossible. Thirdly, some of the avalanches have gone into the sea, and for these all we know is that their runout index has exceeded the runout index at the foreshore. Fourthly, the recording of avalanches is not uniform: A long avalanche is much more likely to have been recorded than a short one.

To complicate matters even more the probability of an avalanche being recorded is dependent on the path. To some extent this is counteracted by the averaging of the frequency described in sections 6.2 and 6.3. The "first way to proceed" discussed in section 4.4 also aims at resolving this problem.

The problem with the sea avalanches is a little technical but not very difficult. It has been solved by "spreading the avalanches over the sea" according to the distribution of the runout indices of the land avalanches. The other deficiencies will be addressed in sections 4.3 and 4.4.

4.2 Density and distribution functions

Using a statistical estimation procedure known as *kernel estimation*, the combined history of the standard slope can be used to estimate a probability distribution of runout indices. Figure 5 shows the estimated density function and the distribution function with a histogram of the runout indices superimposed on the density function. Kernel estimation can be thought of as a smooth histogram of the data. The reason that the density function lies above the histogram near the right end is that in the histogram the sea avalanches are recorded as having stopped at the coastline.

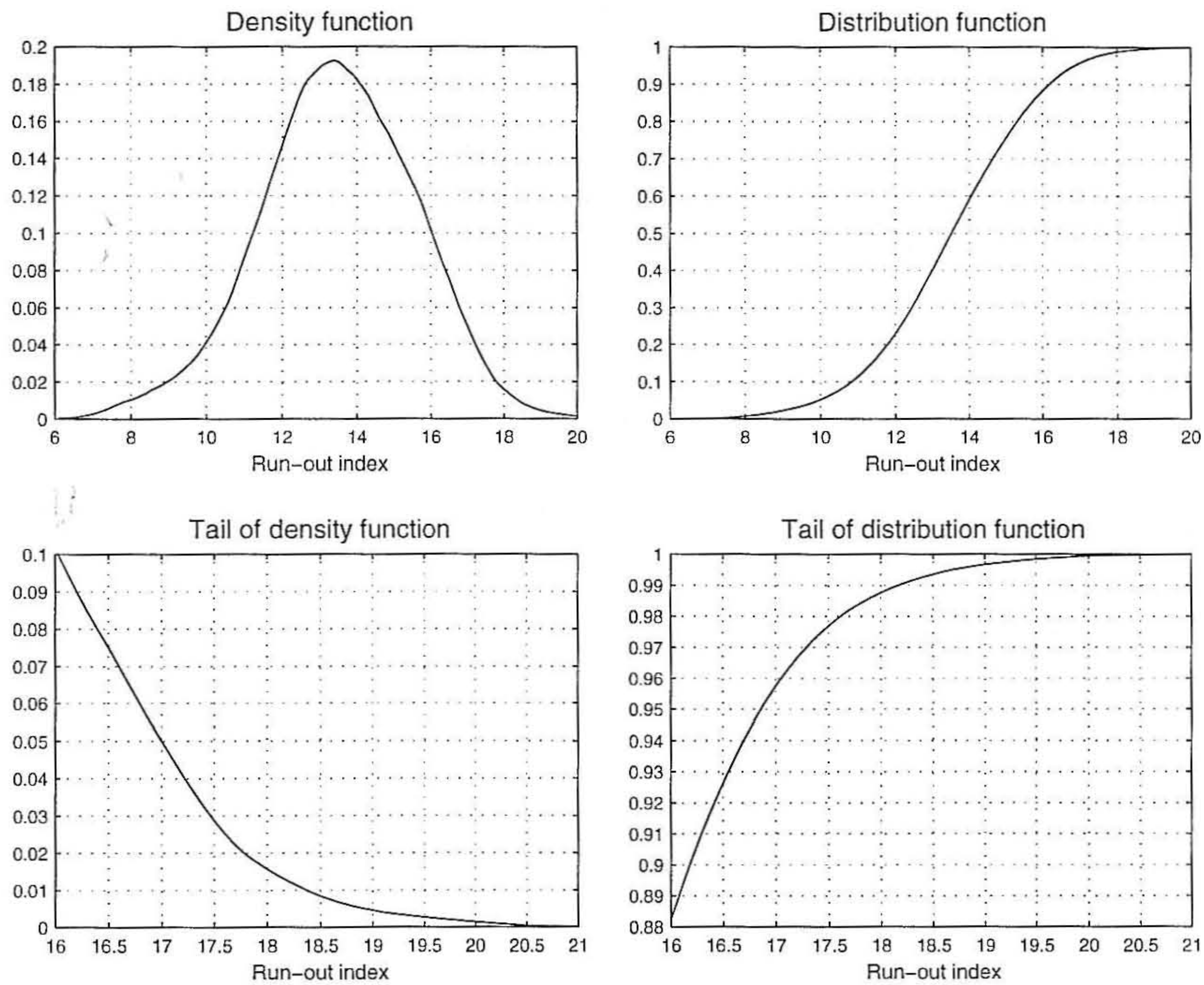

Figure 5. Runout index density and distribution functions for 197 Icelandic avalanches.

Table 3 shows, for a range of runout indices, the *exceedance probability*, or probability of a "random recorded avalanche" reaching the index. If the set of 197 avalanches can be considered to be a random sample from some population of avalanches then a "random recorded avalanche" is an avalanche selected at random from this population.

Runout index, r	Probability of avalanche reaching r, $P(r)$
12.0	77.1%
12.5	69.0%
13.0	59.8%
13.5	50.3%
14.0	40.9%
14.5	32.2%
15.0	24.3%
15.5	17.5%
16.0	11.8%
16.5	7.4%
17.0	4.3%
17.5	2.3%
18.0	1.2%
18.5	0.7%
19.0	0.3%
19.5	0.2%
20.0	0.1%

Table 3. Exceedance probabilities for runout indices.

4.3 Differences in avalanche frequency

The first and second flaws discussed in section 4.1 are interrelated. The basic assumption we make is that the relative frequency of short and long avalanches, as measured by their runout indices, is the same in all the hillsides. Thus we are really ignoring the second flaw and it follows that the estimated distribution of the previous section can be considered to be global. This implies that the frequency of avalanches going past a point under a particular hillside is given by

$$\text{Frequency at the point} = P(\text{runout index at the point})\cdot R$$

where P is the exceedance probability and R is the "frequency of recorded avalanches". The problem of determining R is dealt with in the next section.

Let us clarify with a simple example. There is a house standing at runout index 17 under a path with 5 recorded avalanches in 50 years. Assume that the probability that an avalanche from the path with a particular runout index was not recorded is the same as the probability that an avalanche with this runout index is missing from the big data set. Then $R = 0.1$ (per year), the frequency of avalanches hitting the house is $0.5\cdot P(17) = 0.1\cdot 4\% = 0.004$ (i.e. 4 per millennium) and the return period is 250 years. In this example we have ignored the fact that because of the shape and direction of the tongue of the avalanche the house might escape an avalanche with runout index higher than 17. This is the tongue effect discussed in chapter 7.

Returning to the second flaw there is at the moment no direct solution. Let us first say that it is not certain that there is a problem. From the investigation of the data made so far it cannot be ruled out that the basic assumption of the last paragraph really holds and that a global runout index distribution does in fact exist. Failing that the obvious way out would be to find some way of grouping the avalanche paths so that all the paths in a group have the same runout index distribution, but different groups have different distributions. This has proved to be a difficult task and attempts of finding simple characteristics that affect the distribution of runout distance have largely failed, although some success has come from looking at low hillsides and high hillsides separately.

Presently we are taking the basic assumption of uniform runout index distribution for granted. For a different course it will be necessary to gather more data on Icelandic avalanches and make further analysis of the data.

4.4 Missing avalanches

Because long avalanches are much more likely to have been recorded than shorter ones, the tail of a distribution calculated from these recordings will be too thick. However, if all avalanches with runout index greater than some specific value are recorded, and we can estimate accurately the frequency of avalanches from the hillside under consideration that reach the specific index, then the extra thickness of the tail will be of no consequence for the estimated avalanche frequency in areas outside the specific runout index. We have been working on the assumption that the number of missing avalanches with runout index higher than 16 is negligible. Recently, some doubt has been cast on this assumption and further investigation of the matter is planned.

If no recorded avalanches in the area under consideration have reached a runout index of 16 then the frequency of avalanches reaching 16 can of course not be estimated directly.

There are two ways to proceed. The first is to estimate the global proportion of avalanches missing at each runout index and thereby a corrected runout index distribution. Then we can in principle use any value as a reference limit. We estimate the local probability of an avalanche being recorded at each runout index between the chosen reference limit and 16, count the number of avalanches that have reached the reference limit and adjust for the missing avalanches. The estimates of the proportion of missing avalanches could be based on the dates of recorded avalanches, the building history under the hillsides or both. An earlier version of our method was based on a mixture of these ideas: The global distribution was corrected based on the dates of the recorded avalanches and locally the building history was used to guess the probability of an avalanche being recorded. In short the resulting method was quite involved and perhaps not very convincing. Nevertheless we believe that this is the right way to proceed when the time is ripe.

The second and simpler way to proceed, is to work with the uncorrected distribution and assume that at each runout index the proportion of missing avalanches is the same globally and locally. In most of the towns under the hillsides of the avalanche collection the settlement has been slowly moving higher up in the hillside during the last half century or so. If the buildings nearest the mountain in the town in question are located at a similar runout index as in the towns in general and if the settlement movement has been similar then this assumption is supported.

In both cases it is recommended that the lowest part of the distribution is not used. The estimate of the proportion missing will probably be very inaccurate at low runout indices, the accuracy improving at higher values. Similarly the assumption that the global proportion of missing avalanches is the same as the local one is more likely to hold at high runout indices. We return to this matter in chapter 6.

One disadvantage of the second method when compared with the first one (correcting the runout index distribution) is that the hazard will be underestimated at places that are closer to the mountain than at index 16. This fault does not in general affect the position of hazard zone limits as these are usually outside index 16. However it is necessary to make some rudimentary correction to the distribution before it is used to calculate the frequency at places that lie uphill from the 16 index.

5. Survival probability

The probability that a person survives when an avalanche hits a house that he is staying in has been estimated using data from the avalanches of Súðavík and Flateyri. These avalanches damaged a total of 32 houses and 93 people were staying in these houses. For each house the stopping position of the avalanche directly downstream from the house was determined from the outline of the avalanche deposit. In some cases the avalanche went unhindered on both sides of the house, making the determination of the stopping point easy, but in some cases it was necessary to guess the breaking effect of the house and other nearby houses. Having estimated the stopping position, a (μ, M/D) pair that explains the stopping point is found on the axis of section 3.2. Then the speed of the avalanche when it hit the house is estimated as the speed of a PCM-simulated avalanche with this pair of parameters. Figure 6 shows the fraction of people killed as a function of speed both by a histogram and by a smooth curve.

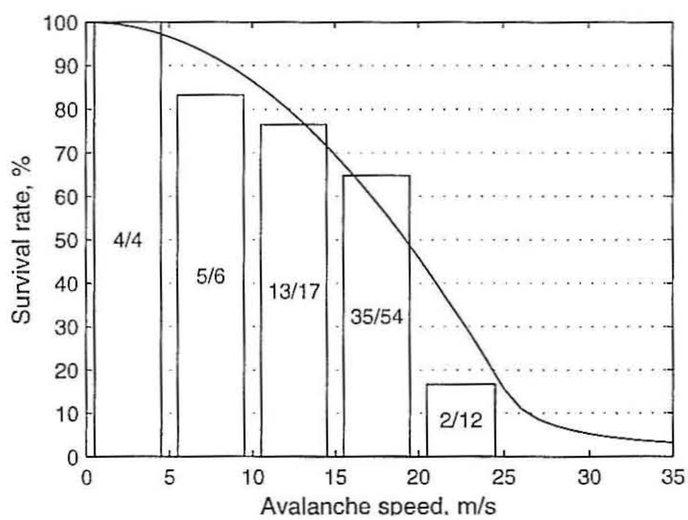

Figure 6. The survival rate in Flateyri and Súðavík according to the avalanche speed. The numbers in the bars show the number of survivors and the total number in each speed group.

The formula for the curve is:

$$s(v)=\begin{cases}1-kv^2 & \text{if } v<v_1\\ \dfrac{a}{v-b} & \text{if } v\geq v_1\end{cases} \tag{1}$$

where $k = 0.00135$, $a = 0.385$, $b = 22.54$ and $v_1 = 24.93$. The curve has been found by assuming that the function is continuously differentiable and has the form (1), and then determining k, a, b and v_1 using maximum likelihood estimation. The form (1) ensures that the survival probability will decrease slowly at low speeds and approach zero asymptotically at high speeds, both being natural assumptions.

Along with the survival probability we work with the death probability which is defined by

$$d(v) = 1 - s(v). \tag{2}$$

This curve is of course only valid if the houses in the area where the method is being applied are of similar strength as the houses hit in Súðavík and Flateyri. We believe that this is the case for most of the houses in the avalanche hazard towns in Iceland.

6. Avalanche frequency

6.1 Single path frequency estimation

In addition to the runout index distribution and survival rate the third basic ingredient in the method is the frequency of avalanches from the hillside under consideration. Contrary to the first two, that are estimated globally and once and for all, the frequency estimate is based on the local history of avalanches. Frequency estimation came up several times in chapter 4 because of its interrelation with the estimation of the runout index distribution. The current chapter will add some details.

We shall limit the discussion to the case when it is assumed that the proportion of avalanches missing is the same globally and locally and the uncorrected runout index distribution is used (see section 4.4). As was mentioned in chapter 4 it is not recommended that all avalanches are counted but instead only those that reach some reference limit r. Assume now that N_r avalanches have reached run-out index r and that the observation period is T. Let Φ be the run-out index distribution function and

$$P(x) = 1 - \Phi(x) \tag{3}$$

(note that $P(x)$ is the exceedance probability, the probability that an avalanche will travel further than x). Then a straightforward estimate of the frequency of avalanches reaching runout index 16 is

$$F_{16} = \frac{N_r}{T}\cdot\frac{P(16)}{P(r)} \tag{4}$$

To take a simple example, assume that 4 avalanches have reached $r = 13$ in 80 years. Then we obtain from Table 3

$$F_{16} = \frac{4}{80}\cdot\frac{12\%}{60\%} = 1/100,$$

so that approximately 1 avalanche will reach the 16-line every century.

There is a trade-off in the choice of r. If r is too high then N_r will be small (or 0) and the frequency estimate will be inaccurate (or useless). But a low value of r will also cause inaccuracy in the frequency estimate because of the inaccuracy in the number of missing avalanches as discussed in section 4.4. This suggests that r should be chosen low enough that a few avalanches have reached r but not much lower.

In practice we have used a few different values, for example $r = 13, 14, 15$ and 16, and obtained from these separate estimates of F_{16}. Then we have either used the average for the final F_{16}-value or looked at the estimates together with other information when determining F_{16}.

If there are enough recorded avalanches from the hillside then the different values obtained for F_{16} can also be used to check the assumption that the proportion of missing avalanches at each runout index is the same in the particular hillside as in the data set in general.

6.2 Hillsides with several gullies.

If avalanches fall mostly from isolated gullies that are deemed to have the same topography, then the frequency can be determined jointly for all the gullies, and this will increase the accuracy in the estimate. If there are M gullies and N_r, T and P are as in (4) (N_r is the total number of avalanches reaching r from all the gullies) then the frequency of 16-avalanches from each gully will be estimated by

$$F_{16} = \frac{1}{M} \cdot \frac{N_r}{T} \cdot \frac{P(16)}{P(r)} \qquad (5)$$

We are on slippery ground here, because if the gullies have different recorded frequencies and the reason is in fact that they are differently shaped (or collect snow differently), then we might be worse off than by estimating the frequency in each gully individually. To aid in this decision, one can apply some statistical test to see if one can reject the null hypothesis, that all the gullies are the same, against the alternative that they are different. We can however not go into the details of this here.

Let us take an example from Neskaupstaður. The following table lists all the recorded avalanches with runout index 13 or higher from 7 big gullies that are above the town.

Gully	Date	Runout index	Maximum width (m)
Bræðslugjár	4.2.1974	13.0	220
	1.2.1936	15.2	130
	20.12.1974	15.5	415
Miðstrandarskarð/Klofagil	20.12.1974	14.9	270
	1.1936	14.3	130
	21.3.1989	13.3	60
Ytra- and Innra Tröllagil	27.12.1974	13.4	190
	3.1920	13.4	140
	1894	15.0	
Urðarbotnar/Sniðgil	28.12.1974	13.5	60
	28.12.1974	13.0	60
Drangaskarð/Skágil	4.2.1974	13.0	220
	20.12.1974	14.0	390
	24.1.1894	15.4	390
Nesgil	4.2.1974	13.1	90
	21.3.1989	13.4	130
	3.1966	14.4	120
	4.2.1974	15.0	180
Bakkagil	4.2.1974	13.3	70
	21.3.1989	13.5	100
	3.1966	14.0	150
	20.12.1974	15.9	260

Table 4. Long recorded avalanches from 7 gullies in Neskaupstaður

The average maximum width of the avalanches is 180 m but if we just take the avalanches that reach runout index 14 we obtain an average maximum width of 244 m.

Let us emphasise that the purpose of this example is to demonstrate the use of the method and we have neither visited Neskaupstaður for this study nor checked the topography of the gullies in details. But a quick inspection of a map indicates that the gullies are all similar. Now set the observation period to

110 years (there is a recorded avalanche in 1885 in Neskaupstaður that does not qualify for entry in Table 4). From (5) with $M = 7$ and Table 4 we obtain the following table of r, N_r, $P(r)$ and the estimate of F_{16}. Note that we give F_{16} in percentages, so the values can be interpreted either as the probability that in 1 year there is an avalanche from a particular gully reaching index 16, or as the number of avalanches per century that reach 16.

r	N_r	P_r	F_{16}
13.0	22	59.8%	0.56%
13.5	13	50.3%	0.39%
14.0	11	40.9%	0.41%
14.5	7	32.2%	0.33%
15.0	6	24.3%	0.38%
15.5	2	17.5%	0.17%
16.0	0	11.8%	0.00%

Table 5. The number of avalanches reaching different runout indices in Neskaupstaður and corresponding estimates of the frequency of 16-avalanches from each gully.

The average of the F_{16} values is 0.37 avalanches per century. These values indicate that there is better than average recording of the avalanches at the lower runout indices. This could be explained by the fact that the houses under these gullies are closer to the mountain than the average for the data set. A real hazard zoning project for Neskaupstaður would involve a further study of this.

6.3 Frequency in hillsides

If we are considering a straight hillside where it is deemed that avalanches fall from each part with equal probability a slightly different approach is needed. Assume that the total width of the hillside is W and the estimated width of an avalanche is A. If N_r, T and P are as before the resulting estimate of the frequency at index 16 is given by

$$F_{16} = \frac{A}{W} \cdot \frac{N_r}{T} \cdot \frac{P(16)}{P(r)} \tag{5}$$

If for instance the area is 800 m wide, an avalanche is 400 m wide and 5 avalanches are recorded with run-out index greater than 13 in 50 years of observation then the frequency estimate at 16 will be

$$F_{16} = \frac{400}{800} \cdot \frac{5}{50} \cdot \frac{12\%}{60\%} = 1/100 \text{ per year,}$$

corresponding to a return period of 100 years.

Notice that A is the *average* width of the avalanche or more precisely the width of an *equivalent* rectangular avalanche where the meaning of *equivalent* is admittedly somewhat vague. In fact we have in practice been working with the more easily determined *maximum* width of the avalanche instead of the *average* width. This causes overestimation of the risk and to compensate we pull the calculated risk lines towards the mountain. We explain this better in chapter 7 where we also attempt to determine the necessary amount of pulling.

7. Tongue effect

In section 4.3 we mentioned the possibility that a house is missed by an avalanche that goes further than the house. This effect is demonstrated in Figure 7 where a house at runout index 17 is missed by an avalanche of runout index 18. We are well aware of the effect but so far we have dealt with it in a rather rudimentary fashion.

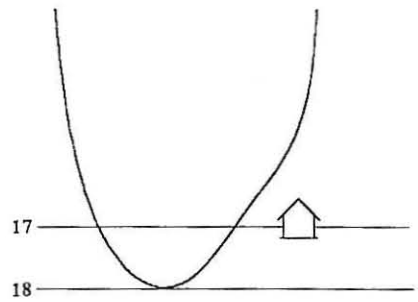

Figure 7. The tongue effect.

We selected a small set of about 7 avalanches with different tongue shapes that were deemed to be representative for the shapes of all the avalanches in the collection. We then calculated the position of several risk lines under the standard slope using the risk model described in chapter 8, assuming that the runout distance was distributed according to the global runout index distribution of section 4.2 and that the shapes were selected at random from the 7. We then repeated the calculation under the assumption that all the avalanches were rectangular and it turned out that equivalent risk lines were about 50 m further away from the mountain. The calculation was done using a few different frequencies and we also checked the result in a few other paths. The results differed a bit, but remained in the neighbourhood of 50 m.

Under straight (gully-less) hillsides we have taken the tongue effect into account by pulling all calculated risk lines 50 m towards the mountain.

Under gullies the situation is a bit more complex since a house that is a little to the side of the main direction from the gully can escape both because of the shape of the avalanche and because the avalanche takes a direction away from the house when leaving the mouth of the gully. On the other hand it is possible that the risk lines should be pulled less than 50 m back directly under the gully because the tip of a gully avalanche is often there. Up till now we have been solving this problem quite heuristically but further investigation of the tongue effect under gullies is planned.

8. Risk model

8.1 Formulas for risk

We now have all the necessary ingredients to present the promised formula for calculating the risk of living or working in a building under an avalanche hillside. The total risk will be the aggregate of the risk from short, medium and long avalanches. Depending on the placement of the building the different length avalanches will contribute differently to the total risk. The long avalanches will be rare, but devastating when they fall. The short avalanches are, however, more frequent but not as harmful (or even totally harmless if the building is not within their reach).

Assume that the building is placed at runout index r_0 and that the largest possible avalanche has runout index 20 (cf. Table 3). Let the frequency of avalanches that reach runout index 16 be F_{16}. From chapter 4 it follows that the frequency of avalanches at any runout index $r \geq 16$ is given by

$$F_r = F_{16} \cdot \frac{P(r)}{P(16)} \tag{6}$$

where P is the exceedance probability given by (3) and F_r is the probability of getting an avalanche with runout index r or greater in a one year period. If r < 16 then (6) will not be accurate unless we are working with a corrected runout index distribution (c.f. section 4.4), otherwise the frequency given by (6) will be an underestimate, and this will lead to the risk being underestimated at the lower r-values.

The risk formula can be put accurately forward using an integral but this is (maybe) not very transparent so we begin by presenting a rather rough approximation that we hope is more evident. We denote the speed of the avalanche in m/s when going past the building with v and let $p(\mathrm{A})$ mean the probability that event A occurs in a one year period. We let $d(v)$ be the death probability given by (1) and (2). If no avalanche can hit the building with a speed greater than 50 m/s we obtain the following approximate formula for the risk of a person that spends all his time in the building:

$$\text{Risk} = p(0 < v \leq 10)\cdot d(5) + p(10 < v \leq 20)\cdot d(15) + \ldots + p(40 < v \leq 50)\cdot d(45).$$

We can calculate the probabilities $p(v_1 < v \leq v_2)$ using (6) and a table of speeds and runout indices calculated with the PCM-model. The more accurate integral formula is

$$\text{Risk} = \frac{F_{16}}{P(16)} \int_{r_0}^{\infty} \phi(r) d(v(r)) dr \tag{7}$$

where ϕ is the runout index density function. This is the formula that we have actually been using, together with the pulling explained in chapter 7.

8.2 Practical experience with the risk model

The model that is described in the previous chapters has been tried out for about ten hillsides in Iceland. Some of these hillsides are Flateyri (an earlier version of the model), Seljalandshlíð in Ísafjörður, Drangagil in Neskaupstaður and Bjólfur in Seyðisfjörður.

The frequency estimates that we have obtained vary widely. They are summarised in Table 6 along with the return period at the acceptable risk line (the line where the risk is $0.3 \cdot 10^{-4}$, cf. section 2.3) and the runout index at the acceptable risk line. The column headed "recorded frequency" is the R of section 4.3 (frequency of recorded avalanches). The table should be considered as giving a rough indication of the values obtained as the values in it have not been thoroughly checked.

Hillside	Frequency at r=16	Recorded frequency	Acceptable return period	Acceptable runout index
Flateyri	1/50–1/30	25%–40%	ca. 2000	19
Seljalandshlíð	1/150	8%	3500	18
Drangagil	1/300	4%	4000	17.5
Bjólfur	1/1000–1/200	0.5%–6%	3500–6000	16.5–18

Table 6. Estimated avalanche frequency in different hillsides.

We also remind the reader of the reservations put forward at the end of section 2.4

9. Concluding remarks

We wish the reader of this report to keep in mind that it is not meant to be a comprehensive description of the risk assessment method and therefore several details are not included. It should also be borne in mind that the report is somewhat hastily put together and should be considered to be a preliminary version of a fuller English report due later. The final report mentioned in the introduction will however be in Icelandic.

We also wish to emphasise that what we have described is not a comprehensive method for avalanche hazard zoning that takes everything into account. The method is designed for assessing the risk caused by avalanches from hillsides that have some recorded history of avalanches. It will not help in identifying starting zones of avalanches. It is not suitable for assessing the risk from slush flows or mud flows. It is not really suited for hillsides where there is no avalanche history although it can be used to put an upper limit on the risk under such hillsides and the method is not suitable in its present state for hazard evaluation of areas that are protected by defence walls or supporting structures.

We wish to end these remarks by pointing out that the method can be calibrated in the following way. For each house, both present and past, in the Icelandic towns, we use the method to calculate the risk. We then determine for each house the length of time that it has been standing and estimate the expected average number of people present in the house during each period of its existence. We can then integrate the risk and find the expected total number of people that would have been killed in the last 120 years (about the age of the towns) based on the risk being as calculated. This number can then be compared with the actual number of fatalities.